Randomness, Emergence and Causation: A Historical Perspective of Simulation in the Social Sciences

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Abstract This chapter is a review of a selection of simulation models, with special reference to the social sciences. Three critical aspects are identified—i.e. randomness, emergence and causation—that may help understand the evolution and the main characteristics of these simulation models. Several examples illustrate the concepts through a historical perspective.

Keywords Simulation models · Randomness · Emergence · Causation · Agent-based models

1 Introduction

The following pages present a selection of computational models and techniques that have been used in the last 70 years and provide an overview of how the field has evolved. In an era of cheap and fast computation, it is particularly important to look back at the history to understand the specific reasons that make current advanced techniques so remarkably relevant to social scientists.

We do not pretend to present a comprehensive overview of simulation modeling techniques and systems, but have selected those that we believe have contributed the most to build current simulation approaches. In so doing, we identify a thread, concerning *randomness*, *emergence* and *causation*, that specifies some of the most relevant characteristics of current techniques. In an attempt to make these comments

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as much visible as possible, they are found under the header "Intermezzo", as they interrupt the flow of the presentation and weave together the different methods.

Before moving forward, a few words need to be spent on these three aspects and why they are so important as to appear as the 'fil rouge' of the chapter. Let us start from *randomness*. Any simulation of a social system has to be able to reproduce elements that appear unpredictably and without any apparent connection to the phenomenon under analysis (or to an outcome variable). The reason is connected to Laplace's demon (see [31, p. 2]): an omniscient intellect having complete knowledge of all the forces and positions of the items composing the natural world as well as unlimited computing possibilities would be able to predict without error all future events. Randomness is a way of accounting for our ignorance of these initial conditions and for our computational limits. That is why a model not taking into account the possibility of unpredictable and/or external events would fall short of capturing the inherent complexity of most phenomena. The second aspect, *emergence*, is tied to the assumption that social systems are complex [29, 50]. When this assumption holds for the computational simulation that models a social system, then uncertainty, ambiguity, and unpredictability are key features of that research effort. Some have argued that a successful simulation is one that presents the modeler with counter-intuitive and surprising results (see, e.g., [54]). We do not subscribe to this view, because it is too radical and only fits certain types of simulations. However, we can certainly support the idea that simulating a social system means to allow for a true intellectual enquiry, where results are not entirely discernible by simply looking at the code. On this respect, "a model is *not* a model" (opposite to what some argue in a recent editorial, see [78]). The third aspect is that of *causation*. One of the defining aspects of a computational simulation refers to the mechanisms that specify how its component parts behave. The interactions of these parts are reflected in the values taken by the aggregate variables describing the system that, on their turn, impact the single components. The components and the aggregates are thus linked by up- and downward relations. In a social system, both causal directions need to be present to explain most phenomena. The history of computational simulation in the social sciences has always bounced back and forth between these two levels, and settled on recent techniques that could account for both.

The methods we are going to present are very heterogeneous. Some of them are specified at the level of the individual, others are aggregate. Some of them require interactions between agents, others don't. Some of them are deterministic, others contain random elements. However, all of them share the same two characteristics: (a) objects (individuals or quantities) are reduced to a finite number of idealized types (that may vary in quality); (b) objects are specified by relations partially or fully connecting them together. The historical review that we are going to present will show how the two mechanisms that have governed the evolution of these methods are indeed the identification of basic units of analysis, at whatever level they are defined, and the determination of the mechanisms that connect them. The solutions that have been proposed to these two questions have led to the development of several simulation methods.

Now we come to the structure of the chapter. In Sect. 2, we review some computational experiments involving early computers. In Sects. 3 and 4 we respectively review System Dynamics and Discrete-Event Simulation, two methods of inquiry considering the aggregate behavior of a system. Then, we review Microsimulation techniques in Economics and Political Science in Sect. 5. Section 6 covers Cellular Automata while Sect. 7 introduces Agent-Based Models. Section 8 wraps up the main conclusions.

2 Experiments with Early Computers

2.1 ENIAC

One of the first electronic computers—the ENIAC, Electronic Numerical Integrator and Computer—was built at the beginning of 1945 at the University of Pennsylvania in Philadelphia [105, p. 125]. In the spring of 1946 at Los Alamos, Stan Ulam suggested that ENIAC could be used to resuscitate some statistical sampling techniques that "had fallen into desuetude because of the length and tediousness of the calculations" [105, p. 126]. He discussed the idea with John von Neumann, who sent, on March 11, 1947, a letter to the leader of the Theoretical Division of the Los Alamos National Laboratory, Robert Richtmyer, with "a detailed outline of a possible statistical approach to solving the problem of neutron diffusion in fissionable material" [105, p. 127]:

> The idea then was to trace out the history of a given neutron, using random digits to select the outcomes of the various interactions along the way. […] von Neumann suggested that […] "each neutron is represented by [an 80-entry punched computer] card …which carries its characteristics," that is, such things as the zone of material the neutron was in, its radial position, whether it was moving inward or outward, its velocity, and the time. The card also carried "the necessary random values" that were used to determine at the next step in the history such things as path length and direction, type of collision, velocity after scattering–up to seven variables in all. A "new" neutron was started (by assigning values to a new card) whenever the neutron under consideration was scattered or whenever it passed into another shell; cards were started for several neutrons if the original neutron initiated a fission [41, p. 133].

This led Nicholas Constantine Metropolis and Stanislaw Ulam to introduce, in 1949, the name of *Monte Carlo method* [106] for a statistical sampling method:

> I (Metropolis) suggested an obvious name for the statistical method—a suggestion not unrelated to the fact that Stan had an uncle who would borrow money from relatives because he "just had to go to Monte Carlo" [105, p. 127].

The name of Monte Carlo method is nowadays generally used to denote a rather heterogeneous array of techniques for solving mathematical problems by:

- reducing their solution to the computation of an expectation with respect to a random variable and

- approximating this expectation with the empirical average based on a sample of realizations of the random variable.

The simplest example is the integration of a function on a bounded domain.

Example 1 (*Monte Carlo integration*) The aim is to compute the integral of a function f defined on a bounded domain that we identify, without loss of generality, with the unit interval $[0, 1]$. The integral is $\int_0^1 f(x)\,\mathrm{d}x$. A solution is to remark that a random variable X uniformly distributed over the interval $[0, 1]$ has probability density function 1, so that $\int_0^1 f(x)\,\mathrm{d}x = \mathbb{E}f(X)$. This means that, if we have a sample $\{x_1, \ldots, x_n\}$ of realizations from X, we can approximate $\int_0^1 f(x)\,\mathrm{d}x = \mathbb{E}f(X)$ through $\frac{1}{n}\sum_{i=1}^n f(x_i)$.

Modern statistical sampling methods largely predate the Monte Carlo method.

Example 2 (*Buffon's needle*) An often-misquoted antecedent is Buffon's needle, a mathematical problem requiring to compute the probability that a needle of length ℓ, randomly cast on a floor with equally-spaced parallel lines at a distance d, lands on a line. The problem was presented by Buffon in 1733 at the Académie Royale des Sciences of Paris (see [16], where the problem is presented but not solved). A solution was described in [17, pp. 100–105] (see also [30, pp. 359–360]). The original aim of Buffon was to look for an explicit solution. The probability itself is $2\ell/\pi d$ (when $\ell \leq d$) and this explains why several authors have performed the experiment repeatedly to provide, through an empirical approximation to the probability, an approximation to π. Despite the problem lends itself to a sampling solution, there is no evidence that Buffon ever tried to do that. Notwithstanding this, many authors (among which, e.g., [95, p. 120]) have written that Buffon proposed a sampling solution: what probably has misled them is the fact that [33] and [34, pp. 170–171] presented the needle problem together with another problem that Buffon studied with 2048 trials (this is confirmed by the fact that [95, p. 120] states that Buffon tried 2048 tosses in the needle problem). Other authors (see [142, p. 117]) have attributed the sampling solution to [30, p. 360]: despite Laplace is indeed talking about the limit for a large number of draws, he is probably making a reference to a frequentist argument rather than to a real sampling solution to the problem. However, the discussion in [33] (reprinted in [34, pp. 170–171] and [83]) suggests that the sampling approximation of π through Buffon's needle was in use as early as 1855. A search of the literature leads to an even older example, namely [159] (see also [127]).

2.2 Intermezzo

As no computer-based method for building random numbers was (and still is) known, John von Neumann went on to study algorithms to generate *pseudo-random numbers*, i.e. numbers with characteristics similar to those of random numbers. A famous but somewhat trite quotation is:

> Any one who considers arithmetical methods of producing random digits is, of course, in a state of sin. For, as has been pointed out several times, there is no such thing as a random number—there are only methods to produce random numbers, and a strict arithmetic procedure of course is not such a method [154, p. 36].

The interpretation that is generally given to this sentence is inaccurate. Von Neumann was not being skeptical, as often interpreted, of the usefulness of pseudo-random numbers. He was just suggesting that "'cooking recipes' for making digits […] probably […] can not be justified, but should merely be judged by their results" [154, p. 36]. This is the main way in which *random-number generators* (*RNG*, though a better name would be *pseudo-random-number generators*, *PRNG*) are evaluated today, through batteries of statistical tests in which their behavior is compared with the theoretical behavior of true random numbers (e.g., [15, 100]). As computer simulation was, at that time, very difficult, random digits were collected in publications among which the famous *A Million Random Digits with 100,000 Normal Deviates* [27], published in 1955.

However, even before and around the construction of ENIAC several computational experiments were being performed using analog computers, and they often used random numbers too.

2.3 FERMIAC

In the 1930s, when he was still in Rome, Enrico Fermi was studying neutron transport: when a neutron (a sub-atomic particle) passes through matter, it can interact with other particles, or it cannot. However, the aggregate behavior of neutrons seemed out of reach. Fermi assumed that each neutron was like an agent whose behavior was dictated by the sampling of some random numbers. He then computed the aggregate results for a large numbers of neutrons on a mechanical calculator:

> Fermi had invented, but of course not named, the present Monte Carlo method when he was studying the moderation of neutrons in Rome [138, p. 221].

According to his student Emilio Segrè (see [105, p. 128]), Fermi kept the technique secret and used it to solve several problems. His colleagues were often astonished by the precision of his computations. These back-of-the-envelope calculations contributed to create the myth of so-called *Fermi estimates*.

In the late 1947 the ENIAC (see Sect. 2.1) was moved at the Ballistics Research Laboratory in Maryland. During the inactivity of the ENIAC, Fermi built an analog simulator of neutron transport, called the FERMIAC (a pun on ENIAC; an image can be found in [105, p. 129]). In the FERMIAC, neutrons were modelled as agents in a planar region, whose behavior was affected when a material boundary was crossed.

2.4 MONIAC

In 1949, at the London School of Economics (LSE), Bill Phillips (Alban William Housego "A. W." "Bill" Phillips, who later introduced the Phillips curve) built an hydraulic machine called *Monetary National Income Analog Computer* or *MONIAC* (see [116]). The name was a pun on "money" and "ENIAC". It was a system of tanks and valves through which water flowed, in a simulation of money circulation in the UK economy. The example of the MONIAC looks very distant from the ones we will see below, but it contains the features outlined above: here the tanks are the objects; the valves are the relations connecting the objects.

Both the FERMIAC and the MONIAC were examples of *analog* or *analogue computers*, i.e. machines using physical (electrical, mechanical, or hydraulic) phenomena expressed in terms of variables measured on a continuous scale to model a phenomenon. Analog computers were very common when no other computing method was available. The difference with respect to digital computers is that the latter use information stored in discrete form: the earliest digital computers were *program-controlled*, i.e. they were programmed by modifying the physical structure (plugs, wires, etc.) of the machine; modern computers are *stored-program*, as the program is stored in memory, without hardware modifications.

3 System Dynamics

System dynamics (*SD*) is an approach to the dynamical study of systems composed of objects in interaction. The idea of SD is to model the change over time of some quantities through feedback loops, accumulation of flows into stocks, and identification of inflows and outflows. The system is generally represented first graphically as a diagram and then mathematically as a system of differential (or difference) equations, that are then solved numerically by a computer program. Its central insight is the fact that the structure connecting the components is sometimes more important than the components themselves in determining the behavior of the system. It was founded, as a branch of *systems theory* [149], by Jay Wright Forrester in the 1950s (see, e.g., [53] or the historical accounts in [55, 81]). At the beginning, SD was developed to analyze complex business problems, in connection to the author's position at the MIT Sloan School of Management. It has been applied to several problems ever since. SD goes through a series of steps to transform a verbal description of the phenomenon under scrutiny into a mathematical model (see, for example, [144] for a worked-out example on new product adoption).

Example 3 (*Lotka-Volterra Model*) An early example of a system of differential equations in which the elements appearing in the system can be interpreted as feedback loops is the Lotka–Volterra Model (LVM), developed by Lotka [90, 91, pp. 92–94] and Volterra [151, 152] in their seminal work. The LVM is a predator-prey model describing the dynamics of two species—i.e. predators and preys—interacting

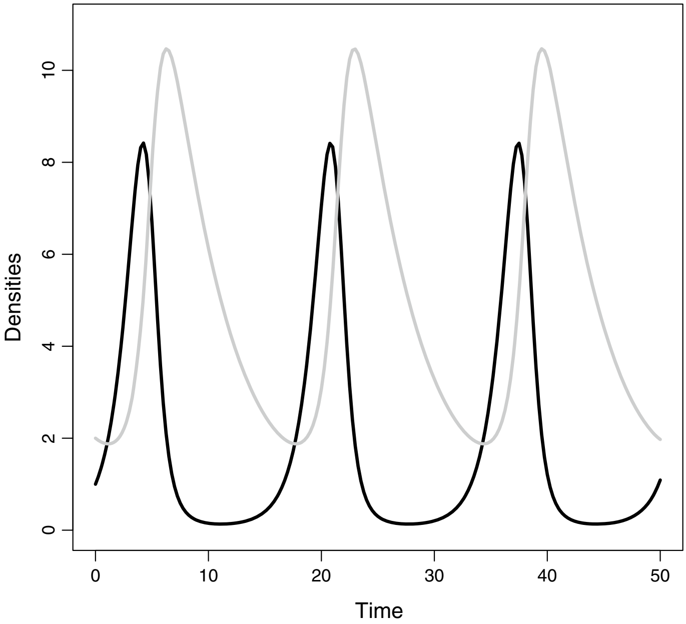

Fig. 1 Population densities of the two species in the Lotka–Volterra model (predator in grey, prey in black)

in an ecological environment. The presence of feedback loops is made clear both by Lotka (see the graphical representation in [90, pp. 411–412]) and by Volterra:[1] "conviene […] di schematizzare il fenomeno isolando le azioni che si vogliono esaminare e supponendole funzionare da sole, trascurando le altre" [152, p. 31]. A SD approach is in [36], while the final behavior of the system is illustrated in Fig. 1.

Example 4 (*The Limits to Growth*) The Club of Rome is a think tank founded in 1968 in Rome as "an informal association of independent leading personalities from politics, business and science, men and women who are long-term thinkers interested in contributing in a systemic interdisciplinary and holistic manner to a better world." Their 1972 book *The Limits to Growth* [104] used SD to study the world economy and population, and raised considerable interest and concern about their sustainability.

Example 5 (*The Lorenz system*) In 1963, Edward Norton Lorenz studied atmospheric convection through differential equations (see [89]). He realized that a small change in the initial conditions could have long-term effects on the behavior of the system. The idea is to take two starting points on two nearby trajectories: moving along them, they will eventually diverge. Similar insights had already been advanced

[1] In English, "it is more effective […] to schematize the phenomenon by isolating the actions that one wants to examine and assuming they behave independently, irrespectively of the others" (our translation).

by Henri Poincaré in 1890 while studying the three-body problem and by Jacques Hadamard while studying motion on surfaces of negative curvature, but had little impact on the literature. Lorenz's discovery, instead, sparked a small revolution. It led to the identification of so-called *deterministic chaos, chaos theory* or, simply, *chaos*, i.e. sensitivity to initial conditions in deterministic systems (often called *dynamical systems*). Lorenz coined the term *butterfly effect* for this phenomenon. An oft-quoted sentence is taken from the title of Lorenz's talk at the 139th meeting of the American Association for the Advancement of Science in 1972: "Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?"

Instances of chaos in models from several domains, and SD among them, were described. As an example, in 1986, Erik Mosekilde and Javier Aracil received the Jay W. Forrester Award for their work on chaos in SD.

Example 6 (*A Sound of Thunder*) In the June 28, 1952, issue of Collier's magazine, a science fiction short story by Ray Bradbury was published under the title *A Sound of Thunder* [13]. It described a time travel into the past whose impact on the future goes awry because of a butterfly (no spoilers). This story is sometimes miscredited with the origin of the name butterfly effect but, despite being a wonderful example of the very concept, it had no bearing on its development.

3.1 Intermezzo

While chaos is extremely important from a theoretical point of view, its relevance in real examples is difficult to work out:

> An essential point made by Poincaré is that *chance* and *determinism* are reconciled by long-term unpredictability. Here it is, in one crisp sentence: *A very small cause, which escapes us, determines a considerable effect which we cannot ignore, and we then say that this effect is due to chance* [130, p. 48; emphasis in the original].

The problem with chaos is that the dependence on the initial conditions makes difficult to forecast the future of the system, as initial conditions are always observed with a small error. This is why chaotic dynamical systems may be modelled as *stochastic processes*:[2]

> En dernière analyse, le hasard réside donc [...] dans l'œil de l'observateur. [43, p. 14]

For us, what matters most is that chaos is a property of the system that is not shared by its components when considered in isolation. Properties like this are called *emergent*:

> The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe. [...] The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. [...A]t each level of complexity entirely new properties appear. [...] Psychology is not applied biology, nor is biology applied chemistry. [...T]he whole becomes not only more than but very different from the sum of its parts. [3, pp. 393–395]

[2] In English: "So, in the end chance lies [...] in the eye of the observer" [44, p. 4].

Emergent properties arise when the system as a whole displays a behavior that is not explicit in its single components. As put forth in [3], a system of interacting quantities/agents is not only more than the sum of its components, it is different from their sum. We will see below some examples of *emergence* (some authors use *supervenience* for a related concept).

For the moment we review some of the history of the concept. As remarked in [79, p. 49], one of the first disciplines to embrace emergence as its central phenomenon was Economics, through the work of Adam Smith:[3]

> [E]very individual […] neither intends to promote the publick interest, nor knows how much he is promoting it. […H]e intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. […] By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it. [141, p. 35]

The philosopher John Stuart Mill wrote, when dealing with failures of the principle of the Composition of Forces:

> The chemical combination of two substances produces, as is well known, a third substance with properties entirely different from those of either of the two substances separately, or of both of them taken together. Not a trace of the properties of hydrogen or of oxygen is observable in those of their compound, water. […W]e are not, at least in the present state of our knowledge, able to foresee what result will follow from any new combination, until we have tried it by specific experiment. [107, pp. 426–427]

The metaphor of water was a recurrent one in the work of early emergentists [12, p. 37]. In Biology, Thomas Henry Huxley, in the book [71, pp. 16–17], introduced the idea that "there is no sort of parity between the properties of the components and the properties of the resultant": he used the "aquosity" of the oxide of hydrogen (i.e. water) as a comparison for the "vitality" of living systems, and he admonished that those who say that "the properties of water may be properly said to result from the nature and disposition of its component molecules" are "placing [their] feet on the first rung of a ladder which, in most people's estimation, is the reverse of Jacob's, and leads to the antipodes of heaven." Another oft-quoted antecedent involving a different discipline is the recognition by the French sociologist Émile Durkheim that social facts cannot be reduced to the agents that are involved in them:[4]

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[3] The sentence is often misquoted replacing the obsolete "publick" with the more modern "public".

[4] The article containing this quotation became the Préface of the second edition of *Les Règles de la méthode sociologique* [39], and is generally quoted as such (despite the article is antecedent); the sentence is not in the first, 1895, edition. In English:

> The solidity of bronze lies neither in the copper, nor in the tin, nor in the lead which have been used to form it, which are all soft or malleable bodies. The solidity arises from the mixing of the two. The liquidity of water, its nutritive and other properties, are not in the two gases of which it is composed, but in the complex substance they form by coming together. […Social facts] reside in the society itself that produces them and not in its parts, namely, its members. [40, pp. 39–40]

It is difficult to say whether Durkheim was aware of Huxley's example, but he was surely well acquainted with the work of Huxley's friend, Herbert Spencer (see [45]), on social organisms. By

> La dureté du bronze n'est ni dans le cuivre ni dans l'étain ni dans le plomb qui ont servi à le former et qui sont des corps mous ou flexibles; elle est dans leur mélange. La fluidité de l'eau, ses propriétés alimentaires et autres ne sont pas dans les deux gaz dont elle est composée, mais dans la substance complexe qu'ils forment par leur association. [...Les faits sociaux] résident dans la société même qui les produit, et non dans ses parties, c'est-à-dire dans ses membres. [38, p. 9]

Several other historical examples are in [102, pp. 63–64, 863]; outside Biology, the economist Elinor Ostrom quoted this book [120, p. 44] as one of her major sources of inspiration.[5]

4 Discrete-Event Simulation

Discrete-Event Simulation (*DES*) is a kind of simulation developed in the 1950s that:

> utilizes a mathematical/logical model of a physical system that portrays state changes at precise points in simulated time. Both the nature of the state change and the time at which the change occurs mandate precise description. Customers waiting for service, the management of parts inventories, or military combat are typical application domains for discrete event simulation. [115, p. 370] or [114, p. 149]

Strictly speaking, the name DES denotes (almost) any simulation taking place in discrete time, but this has several consequences on how it is performed. The system has finitely many components, with finitely many states. These components interact through events having no duration. In general, the state of the system is described by a *state variable*.

Example 7 (*Queueing Systems*) The most classical example of a DES is a queue. As an example, individuals from a *calling population* arrive at random times in front of one or more *servers*, servicing them in *FIFO* (*first in first out*) order with random serving times; if all servers are busy, a *waiting line* creates and its length is the state variable. Several queueing systems are easily solvable, others are not, and require simulation to be solved. An example may be found in [6, Sect. 2.1]: the random values for the arrival and service times are collected in tables, and the number of customers in the system can be obtained combining these values through the rules of behavior of the queue.

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the way, the metaphor of water is used in [45, p. 96] to describe the shorthand system developed by William George Spencer, Herbert Spencer's father.

[5] These pages by Mayr contain some mistakes. First, the book of Lloyd Morgan cited by Mayr is probably the one from 1923 [111], not from 1894, as emergence starts appearing in his work from 1912 (see [12, p. 59]). Second, the quotation just after that is not by Morgan but is taken from the book [121, p. 72] where it is used to illustrate the reasoning in [112, p. 59].

5 Microsimulation

5.1 Microsimulation in Economics

Microsimulation was introduced in 1957 by Guy Henderson Orcutt in [119]. Here is a definition adapted from the one provided by the International Microsimulation Association:

> Microsimulation refers to a wide variety of modeling techniques that operate at the level of individual units (such as persons, firms, or vehicles), with rules applied to simulate changes in state or behavior. These rules may be deterministic or stochastic, with the result being an estimate of the outcomes of applying these rules, possibly over many steps involving many interactions. These estimates are also at the micro level, allowing analysis of the distribution of the outcomes and changes to them, as well as the calculation of any relevant aggregate. [48, p. 2142]

Microsimulation bears resemblances with agent-based modelling (see Sect. 7) but they "have remained very distinct fields in the literature with microsimulation methods drawing heavily on micro-data" [48, p. 2142]. The reliance on micro-data for the construction of rules of behavior—that is generally considered a positive, when not the defining feature, of this method—has somewhat limited the scope of application of microsimulation to situations in which these data are available.

5.2 Intermezzo

What is missing from Microsimulation can be illustrated using the so-called *Coleman's boat* reproduced in Figs. 2 and 3.

The simplest form of this diagram (see [24, pp. 8 or 10]) is shown in Fig. 2. It illustrates the causal paths between micro- and macro-level phenomena: a macro-level cause influences agents at a micro-level and this in turn influences the macro-

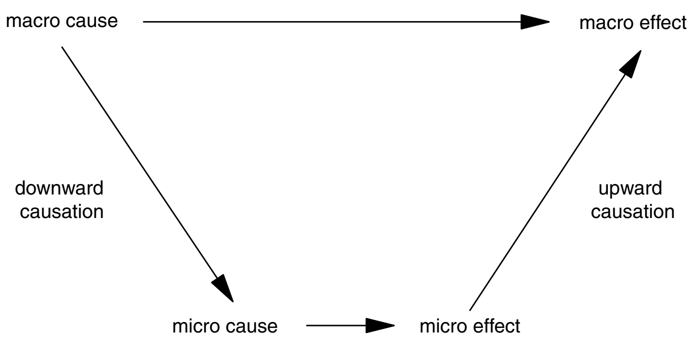

Fig. 2 Coleman's boat as represented in [24, pp. 8 or 10]

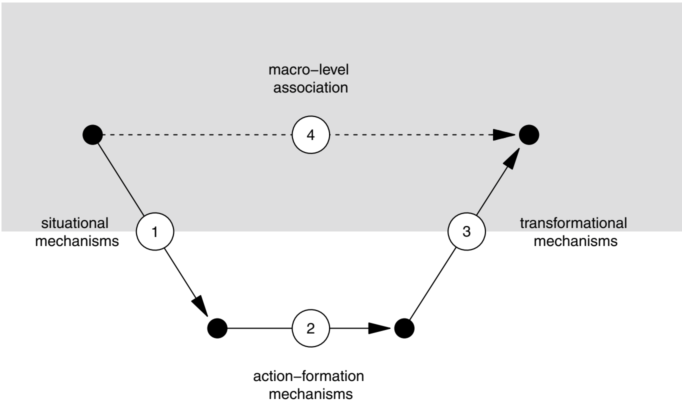

Fig. 3 Coleman's boat as represented in [65, p. 59]

level. Here, "the macro level is an abstraction, nevertheless an important one" [24, p. 12]. A slightly different representation, from [65, p. 59], is in Fig. 3. The grey area represents the macro level, while the white one represents the micro level. Applications to simulation methods are in [146, p. 35], [65, p. 59] and [66].

Before turning to the explanation of these graphs, we remark that some authors (see [18, p. 454] and [98, p. 42]) prefer to refer to this representation as a *Boudon-Coleman diagram* (see [126] for a study of the antecedents of Coleman's boat) while others present modifications of the boat without an upper macro-macro path [23, p. 1322].

The mechanisms "by which social structures constrain individuals' action and cultural environments shape their desires and beliefs" [65, p. 59] represented by path 1 are called *situational*. They represent *downward causation*. *Action-formation* mechanisms (path 2) "[link] individuals' desires, beliefs, etc., to their actions" [65, p. 59]. The mechanisms "by which individuals, through their actions and interactions, generate various intended and unintended social outcomes" [65, p. 59] are called *transformational* (path 3). They represent *upward causation*. Path 4 does not represent causality as "explanations that simply relate macro properties to each other […] are unsatisfactory" [65, p. 59]. One can iterate the "boat" over time: at each step of the simulation, there is an upward causation path from the agents towards the macro-level, and a downward causation path from the macro-level to the behavior of the agents.

The link of emergence with Coleman's boat is that emergent phenomena can generally be identified with macro behaviors induced by the mechanisms taking place in the bilges of the boat. Transformational mechanisms are especially relevant as they are the final step through which upward causation generates emergent phenomena. It is no surprise, therefore, that simulation methods built from the characteristics of the single agents may be better at modeling emergence (see [140, p. 230]).

Now, returning to Microsimulation, Economics has at least two mechanisms of individual market coordination that are coherent with both upward and downward causation: *general equilibrium* and *partial equilibrium*. The former takes place when equilibrium between demand and supply is achieved on all markets inside an economy at the same time, and changes in one market affect all other markets. The latter happens when one market is considered in isolation and is supposed not to affect the other markets. Both mechanisms predict that individuals, without coordination but only through *tâtonnement*, select prices achieving a macro-level equilibrium characterized by *market clearing*—i.e. full allocation of goods in any market of the economy. As individuals face these prices, this constitutes a source of downward causation, from the macro level to the micro one. But the application of partial and general equilibria in simulated models has two problems. First, it is not credible that these concepts of equilibrium hold exactly true, as perfect market clearing seems to be the exception rather than the norm. Second, in models representing a proper subset of the economy, it is difficult to imagine quantitative mechanisms of downward causation.

Coleman's boat can also be useful to classify simulation models. Indeed, some authors [58, 92] identify three categories of models:

- Macrosimulations (e.g., System Dynamics, see Sect. 3, Discrete-Event Simulation, see Sect. 4) focus on an aggregate level and operate at the level of the deck of Coleman's boat;
- Microsimulations (e.g., Microsimulation, see Sect. 5.1, Simulation of Voting Behavior, see Sect. 5.3) focus at the individual level and take place in the bilges of Coleman's boat;
- the third category is composed of models in which there is an iteration between the two levels. These are identified with so-called Agent-Based Models (see Sect. 7).

5.3 Early Simulation of Voting Behavior

Some models similar to economic Microsimulations can be found in the early literature on simulation of voting behavior. We include these models in this review because of their accent on agents' heterogeneity.

Pool and Abelson [122] presented a model they had developed for the Democratic Party in the 1960 US presidential campaign, the so-called Simulmatics project. The original model used the positions of 480 types of voters (that were consolidated to 15 in the published paper) on 52 issues. The data for the model were based on over 100,000 interviews in polls collected over 10 years by polling firms. The researchers advised Kennedy that he would benefit from taking a strong stance in favor of civil rights and from openly dealing with his Catholic religious beliefs. The paper was so influential that in 1964 Eugene Leonard Burdick, a political scientist and novelist, wrote a novel, called *The 480* (see [19]), criticizing the fact that the use of computer models made easy to choose strategies to maximize votes and manipulate electors.

In 1965, Ithiel de Sola Pool, Robert P. Abelson and Samuel L. Popkin published *Candidates, Issues, and Strategies: A computer simulation of the 1960 and 1964 presidential elections*, a book describing in detail their model (see [123]). [1] considered a simulation model of voting in the fluoridation referendum (i.e. whether tap water should be compulsorily fluoridated or not). The model had 500 agents behaving according to 51 rules (22 about information processing, 27 on information exchange, 2 for voting behavior).

6 Cellular Automata

Cellular automata (CA, sing. *automaton*) are systems composed of individuals taking on one of a discrete number of states, arranged in fixed cells (hence the name *cellular*) on a grid, interacting according to deterministic rules depending on the neighboring agents' state (hence the name *automata*). A more formal definition is the one in the Stanford Encyclopedia of Philosophy:

> CA are (typically) spatially and temporally discrete: they are composed of a finite or denumerable set of homogeneous, simple units, the atoms or cells. At each time unit, the cells instantiate one of a finite set of states. They evolve in parallel at discrete time steps, following state update functions or dynamical transition rules: the update of a cell state obtains by taking into account the states of cells in its local neighborhood (there are, therefore, no actions at a distance). [10]

They have been used both as specific examples of real-world phenomena and as abstract examples of how complex behavior can arise from simple rules. Note that the rules of behavior of cellular automata are deterministic and fixed. They were first formalized by Stanislaw Ulam and John von Neumann in the 40s, while the former was working on the growth of crystals and the latter on self-replicating systems. The work of von Neumann culminated in the classic [153] (note that the symposium for which the paper was written was held in 1948), while the work of Ulam was published in [148]. However, it was only in the 1970s that CA rose to prominence with the following example.

Example 8 (*Game of Life*) In 1970, in [56], Martin Gardner popularized "a fantastic solitaire pastime" invented by John Horton Conway. This is indeed a cellular automaton with very simple rules:

- each cell can be either occupied by a living creature or empty;
- the creature in the cell dies if it has 1 or 4+ neighbors (resp. of loneliness and overcrowding);
- an empty cell comes to life if it has 3 living neighbors.

The neighbors of a cell are the ones in the *Moore neighborhood*, i.e. the set of 8 cells in contact through a side or a corner with the cell (a *von Neumann neighborhood*, instead, is the set of 4 cells in contact through a side with the cell). The game is generally applied starting from a configuration of activated cells. At the beginning,

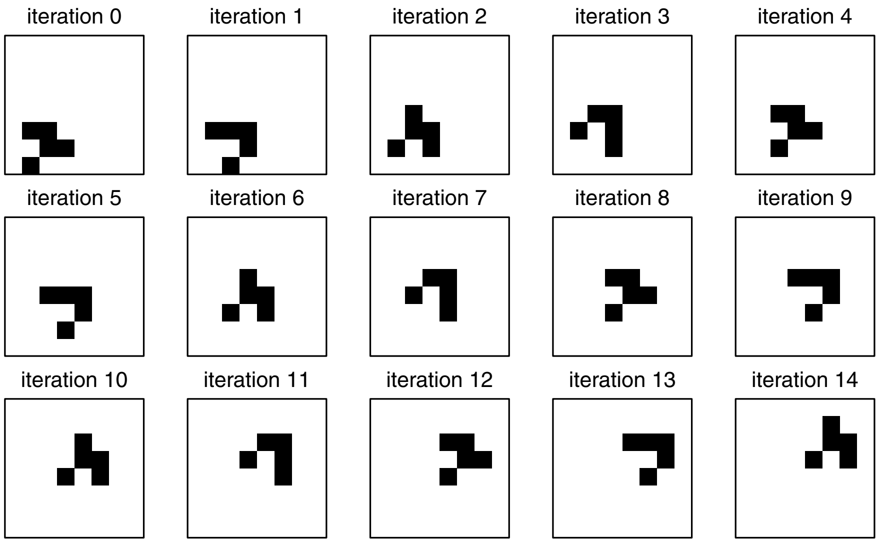

Fig. 4 Game of Life: evolution of a glider on a 8×8 checkerboard

Conway thought that such a system could not create a universe in constant expansion, but he was soon proved to be wrong (see the *glider* in [9, p. 931] or Fig. 4 and the *glider gun* in [9, p. 935]). The number of configurations that have been explored is incredibly large (see [9, Chap. 25]).

Around the same years, Thomas Crombie Schelling introduced a model dealing with *segregation*, i.e. the enforced separation of different ethnic groups in a community. By taking inspiration from James Sakoda, who created a set of so-called checkboard models (see [67] for the detailed story), *Schelling's Segregation Model* [131, 132] showed that a personal slight preference towards a less diverse neighborhood could create in the long run a segregated community.

Example 9 (*Schelling Segregation Model*) Schelling [131, 132] proposed a model in which two types of individuals, say, A and B, are located on a one-dimensional or two-dimensional grid. Some of the cells may be empty. At each step, each individual counts how many in their Moore neighborhood are like them: if the proportion is smaller than a threshold value x, they move to a new position. This position is chosen deterministically: it is the nearest empty position satisfying their threshold. Schelling did not quote explicitly cellular automata in his paper, but to keep the paper inside the framework was compelled to introduce awkward deterministic rules (as an example, individuals choose to move according to a certain order in the grid). However, this is not the version of Schelling's model that is generally used: in the latter, some randomness is generally introduced in the relocation of moving individuals. This small step gets this cellular automaton close to agent-based models (see Sect. 7).

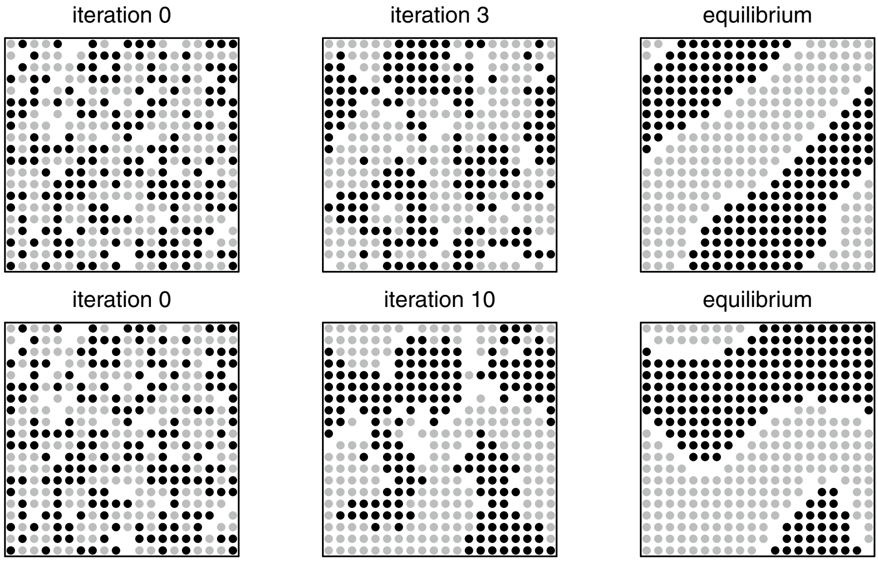

Fig. 5 Schelling Segregation Model for different values of x: the row above shows the case $x = 0.5$, the row below the case $x = 0.9$; the left column displays a random initial configuration that is equal for both values of x; the second column shows the model after 3 (above) and 10 (below) iterations; the last column shows what happens after an arbitrarily large number of iterations

This instance of the model is displayed in Fig. 5. The evolution of the system for different values of x has been characterized:

> Initially the system quickly develops small clusters, but then a slow evolution toward larger clusters follows. [...T]he system evolves toward one big cluster or very few clusters. In the case of $x = 1/2$ the cluster surface tends to form flat surfaces [...] In $x \neq 1/2$ cases the surface is bumpy and irregular [150, p. 19263].

6.1 *Intermezzo*

In this model, segregation is an example of *emergence*. We discuss in the following the implications of emergence and its importance for the model and for the development of simulation in the social sciences.

Example 10 (*Schelling Segregation Model*) In Schelling's model, segregation is an emergent property, as nobody necessarily wants it to take place. Schelling stated this opposition in the famous title of one of his book, *Micromotives and Macrobehavior* [133], whose blurb summarizes the idea as follows: "small and seemingly meaningless decisions and actions by individuals often lead to significant unintended consequences for a large group." The description in the blurb replicates the definition of the upward causation path in Coleman's boat.

Note the striking difference with respect to two economic frameworks that gained traction in the last decades.

On the one hand, in modern Macroeconomics, several models are based on a *representative agent*, i.e. an agent that represents the whole economy (see, e.g., [64]). This became a central element of *Real Business Cycle* (*RBC*) *theory*, first, and, after that, of *Dynamic Stochastic General Equilibrium Models* (*DSGE*) introduced in the two seminal papers [80, 88]. DSGE are, in general, macroeconomic models featuring an economy in general equilibrium; moreover, the models are microfounded, i.e. they are not formulated in terms of aggregate quantities but they derive their behavior by aggregating microeconomic individual models. In the case of DSGE, their behavior can be reduced to that of a representative agent maximizing expected utility. In [62], the authors provide an interesting point of view focused on the causal structure of these dynamic models:

> These types of models are nowadays the most widely used to draw and to evaluate policy claims because they bear the advantage of simultaneously addressing two critical issues about causal structures. On the one hand, under the acceptance of the rational expectation hypothesis, the structure modeled by the RBC/DSGE approach remains invariant under policy intervention because it takes into account the forward-looking behavior of the economic agents. On the other hand, the theoretical structure has an empirical counterpart in which the distinction between endogenous and exogenous variables is eschewed. [62, p. 126]

The emphasis on the representative agent implies that any characteristic of the economy is a characteristic of the agent, and no emergence seems possible (see [79, p. 51]).

On the other hand, in Microeconomics, as well as in other Natural and Social Sciences, some market and non-market interactions, in which tactical and strategic factors are preeminent, are studied through the lens of *game theory*, a branch of economics/mathematics introduced by Oskar Morgenstern and John von Neumann [155] in which agents interact taking into account other agents' reactions. Here emergence is possible as a consequence of strategic interaction between the agents.

These two situations describe a whole spectrum of models, from one in which no emergence is possible to one in which emergence is a consequence of strategic interaction. But Schelling's model is different as emergence is a consequence of the myopic behavior of individuals in a dynamic context. There is no planning at all. (One could even show that, if x is very high, no stable equilibrium is possible: if individuals have strong preferences against diversity, they do not get what they want!)

7 Agent-Based Models

An *Agent-Based Model* (henceforth *ABM*) is a computational model whose unit is the *agent*, an autonomous individual behaving in a given *environment* according to established *rules* [134]. The agent is the unit of analysis and it can be anything the modeler is interested in, from a neutron to a country. Its general features can be characterized by:

- *autonomy*: each agent is modeled independently from the others and it can develop in ways that are not predictable solely by looking at the initial conditions set;
- *interaction*: exchanges with other agents may modify the characteristics of the agent and the way in which it perceives the environment and the self;
- *complexity*: some characteristics "emerge" during the interactions.[6]

Agents interact in a limited space that is usually referred to as *environment*, such that their position can represent either their physical location (see Schelling's Segregation Model in Sect. 6) or their psychological state of mind (see the Garbage Can Model below).

The *rules* are the norms that regulate what happens in the model and are sometimes identified as mechanisms. They can be:

- *behavioral*: they define what each agent should be doing in general and/or as a function of their characteristics;
- *interactional*: they define what happens to an agent and/or to the environment when they interact;
- *time-dependent*: rules may modify agents' characteristics, other rules, or the shape of the environment as time—however defined in the simulation—goes by;
- *developmental*: rules set the conditions for agents (and/or the environment) to change, evolve or die.

The next step we deem appropriate at this point in the chapter is to try and explain the difference between ABM and other simulation frameworks, especially because ABM is the latest and most advanced of all known techniques so far. A classification of simulation models can be based on the following dichotomies:

- the backings of the model can be based on equations or on properties of the objects;
- the approach can be either at the macro- or at the micro-level;
- agents can be homogeneous or heterogeneous;
- rules can be homogeneous or heterogeneous;
- the environment can be either static or dynamic.

While the other concepts have already been explained, it can be interesting to spend a word on backings. Most simulation models involve behaviors that are dictated by equations that connect the different elements of the model. This is clearly true for simulation models working at the aggregate level, but also some microsimulation models are based on actions described through the application of equations on individual-level variables. Some models, however, start from the specification of the characteristics of the objects—be they agents and/or rules—and let them interact more or less freely. A first consideration is that this further increases the distance between the observed behavior of the components of the model and the elements

[6] It is a bit odd to attach this aspect to agents, but we want to highlight that agents can be characterized as complex; see below and [42].

Table 1 A comparison of computational simulation models

	Backings		Approach		Agents		Rules		Environments	
	Equation	Object	Macro	Micro	Homogeneous	Heterogeneous	Homogeneous	Heterogeneous	Static	Dynamic
Early comp.	X		X	X	X		X		X	
SD	X		X			X	X		X	
DES	X		X		X		X		X	
Microsim.	X			X	X		X			X
Voting models	X			X	X		X		X	
CA	X	X		X	X		X		X	
ABM		X	X	X		X		X		X

governing it. Indeed, while the result of an equation is often rather predictable, it is not the case for the interactions of objects possessing their own characteristics and behaving on their basis. For this reason, models whose backings are based on objects often offer more opportunities for the development of those features that are not possessed by their own components but that are born out of the interactions—i.e. exactly emergent properties. This is not to say that equation-backed models cannot exhibit emergence—as the examples above show, they can and they often do—but only that emergence is usually more unpredictable in object-backed models. A second, related, point is that object-backed models are often built using a *bottom-up* approach—i.e. starting from the properties of the objects—and they recover aggregate features only as a result of the interactions. From this point of view, they start from the bilges of Coleman's boat (see Figs. 2 and 3) but they also involve its deck.[7]

From Table 1, it is probably more apparent to understand why ABM is considered the most advanced computational simulation approach as of yet. In fact, by comparing core components of the simulation approaches reviewed so far, it becomes clear how ABM stands at odds with most of them. Of course, the agent-based approach has taken from past simulation techniques, but its comprehensive reach makes it stand as a jump ahead. Omitted from the table and not explicitly mentioned (only cursorily in the introduction) in this chapter so far are the surrounding conditions that make ABM a viable option. We are referring to the surge of computational power and to the possibility that even home computers are capable of performing complicated operations, unthinkable twenty years ago. This means that, for example, having heterogeneous agents in a simulation came at very high costs before the middle of the 1990s, while it is relatively (computationally) cheap to allow them today. This technical hardware and software innovation opened up for the possibility of a different approach to simulation modeling.

In the following pages, we introduce an example concerning the ABM version of the celebrated Garbage Can Model.

Example 11 (*Garbage Can Model*) The *Garbage Can Model* (GCM) of [21] is a model of decision making in an *organized anarchy*, i.e. an organization characterized by the three properties of problematic preferences, unclear technology, and fluid participation.

> [A]n organization is a collection of choices looking for problems, issues and feelings looking for decision situations in which they might be aired, solutions looking for issues to which they might be the answer, and decision makers looking for work. [21, p. 2]

The GCM has been extremely influential in organizational behavior. The model was implemented using one of the earliest computer languages developed by IBM, FORTRAN, and it was not an ABM. Yet, by using an ABM jargon, four types of agents can be identified: (a) problems, (b) opportunities, (c) solutions, and (d)

[7] It is worth noting that ABM can also be backed by equations, better, by a mix of equations and object-based modeling. Actually, we are not aware of ABM that do not have any equation embedded in their coding. The difference of this approach is in the ability to mix and mash both object- and equation-based techniques.

decision makers. The overall goal of the model is to determine whether a formal (hierarchic) organizational structure provides an institutional backbone for problem solving that is better than an informal (anarchic) organizational structure, or not. In the first case, the four types of agent interact following a specified sequence while in the other they interact at random. There are two ways in which participants make decisions in the organization. One is *by resolution*: it happens when problems are solved once participants match opportunities to the right solutions; i.e. when the right combination of the four agents are on the same patch at the same time. The other is *by oversight*: it is when solutions and opportunities are available to participants but no problems are actually solved.

Not all problems are solved automatically, just by having opportunities, decision makers, and solutions available. In fact, all problems have difficulty levels, participants have abilities, and solutions have a certain degree of efficiency. The problem is solved if the match of the participant with an opportunity and a solution overcomes the difficulty of the problem.

The findings of the original model are the following: resolution is not the most common style of decision making; hierarchies reduce the number of unresolved problems but increase problem latency; important and early problems are more likely to be solved.

The model has later been implemented by [51, 52] in ABM form. In the agent-based version of the model, there are three types of structures:

1. *Anarchy*. There is no hierarchy so that abilities, efficiencies, and difficulties are randomly distributed among agents.
2. *Hierarchy-competence*. Abilities, efficiencies, and difficulties increase as one moves up the hierarchical ladder.
3. *Hierarchy-incompetence*. Abilities, efficiencies, and difficulties decrease as one moves up the hierarchical ladder.

Finally, the model implements two modes of (not) dealing with problems (i.e. *flights*):

1. *buck passing*: when one participant has the alternative of passing the decision on a problem to another participant;
2. *postpone*: when problems are kept on hold by participants and eventually solved at an unspecified future time.

The results of the model are summarized as follows by the authors:

> The […] properties point to very interesting features of organizational decision-making. […]
>
> 1. Decisions by oversight are very common, much more common than decisions made in order to solve problems. This result suggests that the rational mode of decision-making is a very rare case. Most decisions are socially induced acts, made with the purpose of obtaining legitimacy by conforming to required rituals.
> 2. If there is a hierarchy, then top executives are busy with gaining legitimacy for their organization by means of decisions by oversight, whereas the bottom line cares about solving problems.

3. Organizations make themselves busy with a few problems that present themselves again and again. So participants have the impression of always facing the same problems. [52, p. 123]

7.1 A Menagerie of Names

ABM have been used in several disciplines: Biology/Ecology [59, 60, 125], Computer Science [37, 117], Sociology [4, 92, 143, 147], Management [2, 110] and Organizational Behavior/Organization Science [7, 49, 136], Political Science [32], Psychology [25, 26] and Cognitive Sciences [93], Population Studies [61], Economics [5, 145] and Finance [157], Transportation Research [94].

As expected, their very general nature implies that ABM can be adjusted in several ways according to the discipline. The different definitions of ABM are exposed in the following parts of the chapter. Since Social Sciences include many different disciplines, we decided to restrict our attention only to a part of these and to compare them with a selection of other disciplines, from the area of Natural Sciences and Techniques (e.g., Physics, Engineering, and Biology). In particular, we dedicate special attention to Economics and Management, highlighting the peculiarities of agents as heterogeneous individuals, with bounded rationality, interacting with each other.

7.1.1 Biology/Ecology

In Biology, an ABM is sometimes called an *Individual-Based Model* (*IBM*), but there is considerable confusion as to the definition. Some authors consider individual-based and agent-based as synonyms (see, e.g., [125, p. 3]). Others reserve the term individual-based when individuals are simpler and rules are formulated probabilistically at the individual level (see, e.g., [11, p. 338]).

Example 12 (*Conservation Biology*) In this branch of biology, and especially in *population viability analysis* (*PVA*)—i.e. the quantitative assessment of risk of extinction concerning a population or a species—most classical models fail to take into account the spatial nature of habitats: spatially dispersed animals, like the giraffe (see [28, 72]), may be at higher risk than commonly thought. *Metapopulation models* were the first to consider several populations interacting in separated locations, but they generally constrain the locations to be discrete. ABM offer a continuous improvement.

7.1.2 Computer Science

In Computer Science, one finds the concept of *Multi-Agent System* (*MAS*, see [37] for a survey and [117] for the relation between MAS and ABM), used to denote a computer system of intelligent agents in interaction inside an environment. The aim is not to study the behavior of agents, but to solve a problem or compute a quantity. An agent is seen here only as a computing entity (and this is why, at odds with most ABM, it must be intelligent). As agents share knowledge or computing power, they approach the solution of the problem. From this point of view, MAS can be seen as a subfield of *Distributed Artificial Intelligence* (*DAI*):

> DAI is the study, construction and application of multiagent systems, that is, systems in which several interacting intelligent agents pursue some set of goals or perform some set of tasks. [158, p. 1]

Another related, but slightly more general concept, is that of *Artificial Adaptive Agents* (*AAA*). This name is often connected with so-called *Complex Adaptive Systems* (*CAS*, e.g., [108]):

> Such a system is *complex* in a special sense: (i) It consists of a network of interacting agents (processes, elements); (ii) it exhibits a dynamic, aggregate behavior that emerges from the individual activities of the agents; and (iii) its aggregate behavior can be described without a detailed knowledge of the behavior of the individual agents. An agent in such a system is *adaptive* if it satisfies an additional pair of criteria: the actions of the agent in its environment can be assigned a value (performance, utility, payoff, fitness, or the like); and the agent behaves so as to increase this value over time. A complex adaptive system, then, is a complex system containing adaptive agents, networked so that the environment of each adaptive agent includes other agents in the system. [68, p. 365; emphasis in the original]

What differentiates CAS from ABM is the emphasis, that is generally lacking in the latter, on adaptivity, but the two approaches are not mutually exclusive (see [118] for an example).

7.1.3 Physics

Physics does not generally require the introduction of agents as sentient and autonomous entities. Optimization provides an example of the difference between Social Sciences and Physics:

> In physics and the natural sciences, maximization typically occurs without a deliberate "maximizer." […M]aximizing behavior differs from nonvolitional maximization because of the fundamental relevance of the choice act. Fermat's "principle of least time" in optics was a fine minimization exercise (and correspondingly, one of maximization). It was not, however, a case of maximizing behavior, since no volitional choice is involved (we presume) in the use of the minimal-time path by light. [139, p. 745]

A notable exception is the area of complex systems, that is at the border of Physics. Outside this area, the generic name of Monte Carlo is used for models that would elsewhere give rise to ABM.

Example 13 (*Why the Brazil Nuts are on Top*) According to physical intuition, when we shake a box of nuts, the largest nuts should go on the bottom while the smallest should float; the evidence suggests that the contrary is true. In [128, 129] the authors build a Monte Carlo model to show how and when this happens.

7.1.4 Economics

In Economics, the most appreciated features of ABM are that they allow for heterogeneity of the agents, both in their types and their characteristics, for interactions taking place in non-trivial, often dynamic, networks [57], and for a wide range of individual behaviors, from perfect foresight to bounded rationality [124].

The name *Agent-based Computational Economics* (*ACE*) is sometimes used to denote the computational study of economic processes modeled as dynamic systems of interacting agents. According to [85, p. 246], an agent refers, in general, to "an encapsulated collection of data and methods representing an entity residing in a computationally constructed world."

ACE may overcome some critiques moved to DSGE models in macroeconomics (see [20, 75, 76]):[8]

> The advantage of the ACE approach for macroeconomics in particular is that it removes the tractability limitations that so limit analytic macroeconomics. ACE modeling allows researchers to choose a form of microeconomics appropriate for the issues at hand, including breadth of agent types, number of agents of each type, and nested hierarchical arrangements of agents. It also allows researchers to consider the interactions among agents simultaneously with agent decisions, and to study the dynamic macro interplay among agents. Researchers can relatively easily develop ACE models with large numbers of heterogeneous agents, and no equilibrium conditions have to be imposed. Multiple equilibria can be considered, since equilibrium is a potential outcome rather than an imposed requirement. Stability and robustness analysis can be done simultaneously with analysis of solutions. [22]

A second difference between ACE and DSGE concerns the agent's expectations, that in DSGE are generally *rational*. This is a mechanism of expectation formation introduced in [113], according to which ex-ante expectations concerning the future value of a variable differ from its real value by a zero-mean random term. This can be justified supposing that agents know the model representing the economy, from which the alternative name of *model-consistent expectations*, i.e. individuals and researchers share the same model of the economy: "Muth's notion was that the professors, even if correct in their model of man, could do no better in predicting than could the hog farmer or steelmaker or insurance company. [...] The professors declare themselves willing to attribute to economic actors at least as much common sense as is embodied in professional theories" [103, p. 53]. Now, this kind of *model-consistent rationality*, according to which agents are able to analyze the economy as economists, cannot be generally assumed in ACE (and more generally in ABM) because of the very way in which models are built.

ACE are widely used in Finance, although, as pointed out by [35, 69, 70, 84], in Financial Economics several features of ABM are not used (interactions taking place over networks, coexistence of several kinds of agents, etc.) and the attention is more focused on the heterogeneity and bounded rationality of consumers.

Another class of models is formed by *history-friendly models* (*HFM*, see [96, 97]):

[8] We refer to [46, 47, 63, 77, 86] for a detailed discussion on the differences between ACE and DSGE.

> "[H]istory-friendly" models (HFMs) of industrial evolution […] are variants of ABMs which aim at capturing in stylized form qualitative theories about mechanisms and factors affecting industrial evolution, technological advances and institutional changes. HFMs consist of three steps: appreciative theories of the history of a specific industry, history-replicating simulations, and history-divergent simulations. In HFMs, model building and calibration are conducted with the guidance of the history. [160, p. 45]

As explained above, an important step in HFM is *calibration*, namely the search for values of the parameters of the model producing an output that is approximately similar to a set of real data.

7.1.5 Management and Organization Research

The status of ABM among the management disciplines is still controversial. In fact, and in spite of the few attempts made so far [49, 109, 134, 137], there is still no clear definition of what ABM are for scholars in the area of Management and Organizational Research (MOR). If one excludes the more engineering-related area of MOR, that is Operations and Supply Chain Management, there are very few examples of ABM (a recent publication guides the reader on how to develop ABM in this field of research; [135]).

In a review of the literature, Wall [156] divides the contributions in two groups, those related to exploration/exploitation and those dealing with differentiation and integration. Models pertaining to the former originate from James G. March's famous categorization of possible opposite decisions an organization usually faces [99] between—to make a very long story short—putting existing resources to work (exploitation) or seeking additional resources (exploration). The latter dichotomy is engrained into very old discourses within the MOR literature, and relates to the basic decision to "make or buy" [82]. One of the most interesting findings of the review—although not discussed openly—is that almost all models are of a special kind: they are NK models (see below). According to another recent study [8], it seems there is a trend in MOR where scholars engage in one of the simplest kinds of ABM while (a) not calling them as such, and (b) de facto establishing a parallel literature. Using NK models to address MOR topics is probably a sign that the field is struggling with concepts such as complexity, emergence, and randomness.

But, outside of this quite remarkable trend, there still are areas of MOR where ABM have appeared. This is the case, for example, of those ABM dealing with routines (e.g., [14, 110]), work team dynamics (e.g., [7, 101]), and, more broadly, with organizational behavior [136].

Example 14 (*Adaptation on a Rugged Landscape*) Given their prominence in MOR, it makes sense to introduce NK models in an example. Before shortly describing the example, it is worth dedicating a few words on this typology of models. They were introduced by evolutionary biologist Stuart A. Kauffman in the late 1980s [73, 74] to study fitness and adaptation. Using Kauffman and Weinberger's words:

> The distribution of the fitness values over the space of genotypes constitutes the fitness landscape. […] The space consists of all 20 N proteins, length N, arranged such that each

> protein is a vertex next to all 19 N single mutant variants obtained by replacing one amino acid at one position by one of the 19 remaining possible coded amino acids. Each protein in the space is assigned some "fitness" with respect to a specific property, such as binding a specific ligand, where "fitness" can be defined as the affinity of binding. An adaptive walk can be conceived as a process which begins at a single protein in the space and passes via ever fitter 1-mutant variants. […] N is the number of "sites" in the model genotype or protein, while K is the number of sites whose alternative states, "alleles" or amino acids, bear on the fitness contribution of each site. Thus K measures the richness of epistatic interactions among sites. [74, pp. 211–212]

Outside of Biology, a modeler could attribute a diversity of characteristics to the two main parameters N and K, preserving their relations, and adapting to the study of different types of "fitness". This was the intuition of Daniel A. Levinthal [87] who was probably the first to introduce the MOR community to NK models. He studied how organizations adapt to different forms (organizational design). In the model, there are N organizational attributes and K other attributes that affect an organization's fitness—i.e. the extent to which interaction affects adaptation. Dependence on initial conditions was one of the main findings of this simulation model.

8 Conclusions

The aim of the chapter was to show the central role played by randomness, emergence and causation for the development of different groups of simulation models. Following the literature, we have outlined a short and necessarily partial history of simulation models, with special attention to the Social Sciences. The models that we have covered are some early works with analog and digital computers, System Dynamics, Discrete-Event Simulation, Microsimulation in Economics and Political Science, Cellular Automata and Agent-Based Models.

To conclude, ABM can be considered the most advanced computational simulation approach so far. Indeed, although this approach has taken from past simulation techniques, its comprehensive reach makes ABM stand as a jump ahead.

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